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# Why Structural Health Monitoring?

- Significant Growth in Urban Living- More than 50% of the World Population now live in cities. This has increased the Importance of Urban Systems safety.
- Deteriorating Infrastructure
- Lifeline Systems Protection Against Natural Hazards
- Visual manual maintenance not reliable and impractical, especially for large, complex structures

The Integrity and Health of a structure, like the human body, need to be monitored constantly to enhance safety and to prolong their lifespan.





# How to Do Structural Health Monitoring?

### Purpose:

- Monitor the system performance
- Detect damage
- Asses/diagnose the structural health condition
- Make maintenance decision

#### **Components:**

- Data Acquisition
- Data Analysis

Damage Interpretation/ Identification

Improvement of

Structural Performance

and Safety

The 1<sup>st</sup> International Workshop on SHM, Stanford University, Sept. 18-20, 1997 The 2<sup>nd</sup> International Workshop on SHM, Stanford University, Sept. 8-10, 1999 The 3<sup>rd</sup> International Workshop on SHM, Stanford University, Sept, 2001



### **Damage Identification Levels**

(Rytter, 1993)

 Damage: Changes in the structure that adversely affect its condition.

- Level 1: Any damage occurs?
   (Determination that damage is present in the structure )
- Level 2: Where is the damage?
   (Determination of the geometric location of the damage)
- Level 3: How severe is the damage?
   (Quantification of the severity of the damage)
- Level 4: Can the system still work? (Prediction of the remaining service life of the structure)





# Key Requirements for Effective, Reliable Quantitative SHM Strategies

 Minimal Dependence on Prior Knowledge of Structure's Dynamics - So that It can Be Used for Linear and Nonlinear Structures

 <u>High Sensitivity</u> - So That It Can Identify Minor and Invisible Damage

 Low Sensitivity - To Measurement Noise That is Inevitable in Real Life Applications





## **Major Developments in SHM Since 1990's**

#### Some Representative Data Interpretation Schemes:

- Modal Based Techniques
- Empirical Modal Decomposition Methods (EMD)/Hilbert-Huang Transform
- Wavelet-Based Techniques

### More <u>Recent</u> Data Analysis Schemes:

- Intelligent Artificial Neural Network Systems
- Support Vector Machines
- Artificial Immune Systems

#### Other Recent Research Issues/Challenges:

- Uncertainty Quantification & Relevance to SHM
- Major Progress in Sensor Technology, Wireless Networked Sensing, Sensor Development





## **A Few Shortcomings of Common Techniques**

- High Dependence on Modeling
- Insensitivity to Local Damage Need of Large Number of Sensors
- Difficulty for On-Line Application
- Vulnerability to Measurement Noise
- Biased Towards Linear, Nonlinear or Hysteretic Structures

Merits and shortcomings of available SHM schemes need to be evaluated to provide a basis for selection of appropriate techniques for future intelligent structures





### **Ultimate Goal?**



An Efficient, Reliable, Economic, Feasible, Integrated, Real-Time System for Predictive Maintenance







### **SHM for Handling Uncertainties**

#### Code-based design

Deterministic design

#### Approach of safety factor (SF)

**Explicit** 

Load safety factor of 1.5

#### Implicit

Conservative decision in all phases of design

#### SF Compensates everything:

- uncertainty in loading
- errors in load and stress calculation
- accumulated structural damage
- variation in material properties
- variation in standards

#### **Challenges**

Selection of probabilistic model

Calibration of model parameters (insufficient data)

What are the other part?

Uncertainty propagation

Significant computational efforts for calculation of response statistics and reliability analysis Probabilistic design

Probability-based design

#### Approach of probability density function

A-basis/B-basis material properties



#### Intelligent structure with <u>Structural Health</u> <u>monitoring</u>can help



#### Detection Capabilities:

- a) Global Techniques- Only infer the existence of damage
- b) Local Techniques Assist in locating the damage
- Extent of Prior Knowledge Required
  - a) **Model-Based** Techniques- Use explicit mathematical descriptions of system dynamics
  - b) **Non-Model Based** Techniques Rely on signal processing of measure responses.

Both Model-Based and Non-Model-Based have been successfully used for damage detection in structural application



### **1.** Model-based Methodologies-

- Damage is regarded as a modification of physical parameters
- Typically rely on parametric system identification using linear, time-invariant models.
- Shortcomings:
  - heavy dependence on system modeling
  - o insensitivity to local damages.
  - o inherent dependence on stationary measurement data.







### **2.** Non-model-based Alternatives-

- Seek to identify damage from changes in dynamic characteristics (e.g, natural freq., mode shapes, etc.)
- Specific patterns or characteristics of vibration response are associated with different structural conditions
- Examples:
  - FT, Short-time FT
  - Time-Frequency Analysis
    - Winger-Ville Distribution approach
    - Empirical Mode Decomposition (Hilbert-Huang Transform),
    - Traditional modal analysis, dynamic flexibility measurements, matrix update methods, Statistical Pattern Recognition, etc.
    - Wavelet Transform





- **3.** Artificial Neural Networks- Used for <u>both model-based</u> <u>and non-model-based</u> damage detection in one of two ways:
  - Their Pattern Recognition capabilities allow the identification of damage using response measurements from damaged and undamaged structures (non-model based approach)
  - Their System ID capabilities enable the estimation of dynamic parameters such as stiffness, mass, and damping (model-based approach)
    - Most published work on System ID has focused on parametric modeling using linear, time-invariant models.
    - However, ANN capability in non-linear function approximation has allowed their use in non-parametric modeling and System ID (e.g. Black Box approach)





## **Focus of This Seminar- Two Research Areas** and Recommendations for Future Work

#### 1. <u>Wavelet-based Methodologies</u>- (Since mid 1990's)

- Wavelet Analysis for Damage Detection(CWT, DWT)
- Wavelet Analysis for detecting sudden & progressive damage and the effect of measurement noise
  - ASCE Benchmark Study Data
  - Experimental Study with DPRI, Japan
- Wavelet packet analysis, and development of Pseudo Wavelets (PWT) for System Identification

### 2. A Special Class of Artificial Neural Networks-

- Development of an *Intelligent Parameter Varying* (IPV) approach- without the limitations of *Black Box*
- Application in System ID for Base Isolated Systems and SHM of linear and nonlinear systems
- Experimental verification of IPV for SHM application







- Development of a Wavelet Based Approach for detecting sudden or progressive damage.
  - Using Continuous and Discrete Wavelet Transform
- Its Application for data from ASCE Benchmark Study
  - Use of Wavelet to locate the damage region
  - Effect of measurement noise
- Development of a Pseudo-Wavelet (PWT) For System Identification
- Experimental Work in Collaboration w/ DPRI, Japan
- Other Applications (Damage in a composite plate due to impact)





• Wavelet is a waveform of effectively limited duration that has an average value of zero. This means that it must be a window function:  $\int_{-\infty}^{+\infty} |\Psi(t)| dt < \infty$ 

Its average value is zero.

$$\int_{-\infty}^{+\infty} \Psi(t) dt = 0$$

It is square integrable or, equivalently, has finite energy.

$$\left\|\Psi(t)\right\| = \int_{-\infty}^{+\infty} \left|\Psi(t)\right|^2 dt < \infty$$

#### **Generation:**

- $\circ$  by explicit formula
- $\circ$  by recursion



#### **Examples of Wavelets:**

### **Daubechies Wavelets**





 $db_3$ 



db<sub>2</sub> Morlet Wavelet











#### Wavelet Transform:

- provides a description of how the spectral content of a signal changes through time.
- breaks up a signal into shifted & scaled versions of the original (mother) wavelet, to extract useful information
- It is a linear transform- Provides time-scale map of the signal, scale parameter is inversely proportional to frequency, has a constant time-frequency window area, has variable time-frequency window resolution

#### Types of Transformation:

- Continuous Wavelet Transform
- Discrete Wavelet Transform







### **Continuous Wavelet Transform**

$$(Wf)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\overline{\Psi}(\frac{t-b}{a})dt$$

$$f(t) = \frac{1}{C_{\psi}} \int_{-\infty-\infty}^{+\infty+\infty} (Wf)(a,b) \Psi(\frac{t-b}{a}) \frac{1}{a^2} dadb$$

$$\circ$$
 a > 0 -----Scaling Factor

- $\circ\ b\in\Re\$  -----Shifting/Translating Factor
- $\overline{\Psi}(\frac{t-b}{a})$  ----- is a translating & windowing function satisfying the *admissibility condition*:

$$C_{\psi} = \int_{-\infty}^{+\infty} \frac{|F_{\psi}(\omega)|^{2}}{|\omega|} d\omega < \infty$$







**Two Basic Operations in Wavelet Transform** 



### **Two Basic Operations in Wavelet Transform**

• Scaling: 
$$\Psi(t) \rightarrow \Psi(\frac{t}{a})$$





W = 0.012

W=0.2247



### **Discrete Wavelet Transform (DWT)**

Signal:  $S(t) = \sum_{j} \sum_{k} \alpha_{j,k} \Psi_{j,k}(t)$ 

The Detail at Level j:

$$D_j = \sum_{k \in \mathbb{Z}} \alpha_{j,k} \Psi_{j,k}(t)$$

The Approximation at Level J:  $A_J = \sum_{j>J} D_j$ 

The Coefficients:

$$\alpha_{j,k} = \int_{\infty} S(t) \overline{\Psi}_{j,k}(t) dt$$

Scaling factor :  $a = 2^{j}$ Shifting factor :  $b = 2^{j}k$ 





 $S = A_1 + D_1$ 

 $j,k \in \mathbb{Z}$ 

 $= \mathbf{A}_2 + \mathbf{D}_1 + \mathbf{D}_2$ 

$$= A_3 + D_1 + D_2 + D_3$$



### **Summary of Wavelet Analysis**

Continuous Wavelet Transform (CWT) || Discrete Wavelet Transform

$$(Wf)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \overline{\Psi}(\frac{t-b}{a}) dt$$
$$f(t) = \frac{1}{C_{\psi}} \int_{-\infty-\infty}^{+\infty+\infty} (Wf)(a,b) \Psi(\frac{t-b}{a}) \frac{1}{a^2} dadb$$

**Details and Approximations** 

$$D_j = \sum_{k \in \mathbb{Z}} \alpha_{j,k} \Psi_{j,k}(t)$$

$$A_J = \sum_{j>J} D_j$$

$$f(t) = \sum_{j} \sum_{k} \alpha_{j,k} \Psi_{j,k}(t)$$
$$\alpha_{j,k} = \int_{-\infty}^{+\infty} f(t) \overline{\Psi}_{j,k}(t) dt$$
$$\Psi_{j,k}(t) = 2^{-j/2} \Psi(2^{-j}t - k) \quad j,k \in \mathbb{Z}$$



**Tree-Structure of Data** 



 $S = A_1 + D_1$  $= A_2 + D_2 + D_1$ 





- Application for SHM are based on the sensitivity of conventional wavelets to singularities in the data.
- A Damage Detection and Location technique was developed. Its sensitivity to damage severity, measurement noise and modeling accuracy was studied.
- Damage Metric Used:

A sudden damage = a sudden stiffness loss \_\_\_\_\_ an abrupt change in acceleration response \_\_\_\_\_ local maxima of wavelet transform modulus (Mallat, 1992)









#### Damage Detection for Sudden and Progressive Damages (Hou, Noori, Raymond)



$$k(t) = \sum_{i=1}^{n} k_i(t)$$
  

$$k_i(t) = \begin{cases} k_{i0}, \text{ if } \operatorname{abs}(x(t')) < x_i^* \ \forall t' \le t; \\ 0, \text{ Otherwise.} \end{cases}$$

$$k_i(t) = \begin{cases} k_{i0}, \text{ if } N(t') < N_i^* \ \forall t' \le t; \\ 0, \text{ Otherwise.} \end{cases}$$





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#### **Damage Detection for Monitoring Progressive Damage**



#### **Comparison with Analytical Results**

Time (sec)	Instantaneous Natural frequency (Hz)								
	1 <sup>st</sup> mode		2 <sup>nd</sup> mode		3 <sup>rd</sup> mode				
	CWT	Modal analysis	CWT	Modal analysis	CWT	Modal analysis			
5	1.2930	1.2932	3.6181	3.6234	5.2357	5.2360			
15	1.2614	1.2619	3.5918	3.5956	5.0542	5.0586			
25	1.2234	1.2230	3.5553	3.5589	4.8774	4.8824			



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### Application for ASCE SHM Benchmark Study Data:

- Objectives of the Study:
  - use the wavelet approach to detect and locate a damage in time and space
  - apply wavelet analysis for on-line monitoring
  - study the sensitivity of the method to damage severity
  - study the effect of measurement noise on the results
  - study the effect of structural modeling and system ID







#### **ASCE Benchmark Study**



ASCE Benchmark Study Data (http://www.tbcad.com/paullam/ascebenchmark.asp).

- Purpose: test and compare various damage identification techniques
- Structure Description: a 4 story, 2 bay X 2 bay steel-frame scale prototype structure
- Damage patterns: removing bracing elements in the structure-4 out of 5 damage patterns proposed by ASCE and additional ones considered (e.g. removing all braces on the 3<sup>rd</sup> floor)

#### Excitations:

- a low-level ambient wind loading at each floor in the y-direction
- a shaker force applied on the roof at the center column position in i+j direction
- Structure Model: 12 DOF Model and 120 DOF Model



	AS	CE Benchm	ark Stu	dy		
Excitation: System Model:		a low-level ambient wind loading at each floor in the y- direction	a shaker force applied on the roof at the center column position in $-\hat{i} + \hat{j}$			
		12 DOF symmetric model	120 DOF asymmetric model	12 DOF symmetric model	12 DOF asymmetric model	
	t=05 sec no damage t=5 sec: - damage pattern 1 (removing all braces in the I <sup>st</sup> story)	Scenario I				
amage pattern	t=05 sec no damage t=5 sec damage pattern 2 (removing all braces in the 1 <sup>st</sup> and 3 <sup>rd</sup> story)	Scenario V				
	t=05 sec no damage t=5 sec damage pattern 3 (removing one brace in the 1 <sup>st</sup> story)	Scenario II	Scenario VII		Scenario VI	
	t=05 sec no damage t=5 sec damage pattern 3 (4) one brace in each of 1 <sup>st</sup> and 3 <sup>rd</sup> story.		Scenario IX	Scenario VIII	Scenario IX	
D	(5) removing all braces in the 3 <sup>st</sup> story;	Scenario III				
	t=05 sec. – no damage t=5 sec: removing all braces in the 4 <sup>th</sup> story t=10sec: removing all braces in the 2 <sup>nd</sup> story t=15sec: removing all braces in the 3 <sup>rd</sup> story	Scenario IV			7	
	t=20sec: removing all braces in the 1 <sup>st</sup> story				Land S	<b>DUTHEAS</b> NIVERSIT

### **ASCE Benchmark Study**

♦ A Typical Damage Scenario-

t=0...5 sec. - no damage t=5 sec. - damage pattern 2 *(removing all braces in the 1<sup>st</sup> and 3<sup>rd</sup> story)* 







Wavelet Analysis for Damage Detection (severe damage)

Damage Pattern 1: Removed all braces, 1<sup>st</sup> story, at t = 5s

Comparison between Fourier spectra before and after damage



 $\triangleleft$ 

ASCE Benchmark Study



Wavelet Analysis for Damage Detection (severe damage)

Damage Pattern 1: Removed all braces, 1<sup>st</sup> story, at t = 5s



Wavelet Analysis for Damage Detection (less severe damage)

Damage Pattern 3: Removed one braces, 1<sup>st</sup> story, at t = 5s

Comparison between Fourier spectra of acceleration at node 15, before and after damage


Wavelet Analysis for Damage Detection (less severe damage)

Damage Pattern 3: Removed one braces, 1<sup>st</sup> story, at t = 5s



**ASCE Benchmark Study** 

### Wavelet Analysis for On-Line Damage Detection

Damage Pattern- Removing: all braces of the 4<sup>th</sup> story at t = 5s + all braces of the 2<sup>nd</sup> story at t = 10s + all braces of the 3<sup>rd</sup> story at t = 15s + all braces of the 1<sup>st</sup> story at t = 20s







### Wavelet Analysis for On-Line Damage Detection

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- -Damage Occurrence;
- -Damage Location;
- -Damage Severity



-Less Model-dependence; -Local Damage Sensitivity; -Robustness to Noise

### **ASCE Benchmark Study**



Wavelet Analysis for <u>Measurement Noise</u> Effect.

Damage Pattern 2: Removed all braces, 1<sup>st</sup> & 3<sup>rd</sup> story, t = 5s



#### Wavelet Level1 Details

Summary of Wavelet Analysis for ASCE Benchmark Study

- A sudden structural damage and the moment when it occurs can be detected
- The region where damage occurred can be identified
- The wavelet approach has potential for an on-line application.
- Effectiveness of wavelet approach depends on the measurement noise level and damage severity.
- Wavelet approach is less model-dependent in the sense only measurement data are required in the analysis.





#### **Pseudo – Wavelet Transform**

- The main capability of Wavelet is due to sensitivity to singularities (sudden stiffness loss). How about if the measurement did not include the moment when a sudden damage occurred?
  - Development of a **Pseudo –Wavelet Transform**-Based technique (PWT) for System Identification
- To use two segments of data, before and after damage, to identify the change of the system parameters:
  - natural frequencies
  - modal damping ratios



#### System Identification Via Pseudo-Wavelet Transform

- Concept of Pseudo-wavelet was developed based on shifting and scaling properties of regular wavelet.
- Pseudo-wavelet Transform was used to successfully identify system natural frequencies and damping ratios.
- Truncated Pseudo-wavelet was proposed to improve accuracy of estimates for MDOF systems and general linear dynamic systems.
- Noise effect was investigated for both measurement & excitation: satisfactory results were obtained.





#### System Identification Via Pseudo–Wavelet Transform







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#### System Identification Via Pseudo-Wavelet Transform





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#### System Identification Via Pseudo-Wavelet Transform

Experimental Validation Using Shaking Table Test Data of A Full-size Two-Story Wooden House (Hou, Noori, Suzuki)











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#### **Identification of Impact Loading on A Composite Plate**







#### Identified impact location (x=0.5. v=0.7 unit)



A composite plate impacted at a point with x=0.5 and y=0.7 unit



#### Wavelet-Packet Based Sifting Process for Damage Detection



Decomposition of response data of a linear 3DOF system and its decomposition by a wavelet-based sifting process



Instantaneous frequency of the third mode for cases of progressive and sudden damage



Comparison with analytical results and results from the Empirical Modal Decomposition method







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### Suggestions for Future Wavelet-based SHM Research

- Further Extension of the wavelet-based technique for a progressive damage pattern;
- Development of wavelet-based indices for health assessment, service life prediction, maintenance decision;
- Comparative study with other approaches, e.g., the empirical mode decomposition, neural network, and Hilbert transform;
- Improvement PWT-based system identification for cases where peaks in the Fourier amplitude response are not well separated.
- Develop a wavelet transform that can detect the instance of damage occurrence under broad-band random excitation.
- Develop a wavelet-based local damage observer using data from a single or very small number of sensors to detect occurrence of damage in a local critical region. (*Progressive damage, Noise effects, Auto-diagnosis algorithms, Warning systems, etc.*)





### Suggestions for Future Wavelet-based SHM Research

- Develop an integrated wavelet-based scheme for SHM of largescale structural systems using local measurements
- Develop a wavelet-based damage isolator to locate damages using measurements from multiple sensors. (*Damage influence* <u>region, Damage isolation algorithms, Optimal sensor placement? etc.</u>)
- Develop an integrated damage estimator to assess local damage severity (Wavelet-based modal analysis for time-varying systems?)
- An integrated wavelet-based EMD scheme and a NN-based online approximation technique, etc.)
- Develop condition-based maintenance guidelines <u>(Structural</u> <u>Health history, health condition indices, etc.)</u>







### Motivation of the Research-

To develop a reliable structural health monitoring and damage detection technique that can detect the:

- Presence
- Location, and
- Time of damage

from recorded structural responses using ANN, traditional modeling, and system identification techniques.

What are the main building blocks of such a technique?

- i. Traditional modeling techniques
- ii. Traditional system identification techniques
- iii. Artificial neural networks







### Challenges of the Research-

- ANN typically involve I/O training to predict the dynamic response of a "healthy" structure to known input excitations.
- This predicted response is compared to the response of the same damaged structure to infer information about the presence, location, and extent of damage.
- Such methodologies, however, <u>may fail to detect</u>:
  - authentic damage if the response of the damaged structure moves beyond the representative domain of the trained neural network.
  - few research has addressed the detection of damage in systems with elasto-plastic and hysteretic restoring force characteristics.





### Outline of the Research Presentation:

- Development of an Intelligent Parameter Varying (IPV)
  Method combining the advantages of Parametric Models with Non-Parametric Capabilities of ANN.
- Application for System Identification of Structures with inherent nonlinearities
- Application in SHM For detecting presence, location and time of damage in linear and nonlinear systems
- An Experimental Study For system identification and damage detection in linear and nonlinear systems.







#### Outcome of the Research - IPV A Unique nonlinear System ID.

- Detects damage in nonlinear structures under seismic excitation
- Integrates capabilities of non-parametric radial basis function ANN, (RFBN) with a traditional parametric model to identify nonlinear, time varying system dynamics (e.g. inelastic and hysteretic restoring forces)
- Provides functional representations of the <u>system nonlinearities</u> without prior knowledge of their constitutive characteristics
- Reveals the <u>evolution of damage</u> through the identification of restoring forces, rather than comparing response characteristics to a "healthy" reference state.
- Uses recorded <u>inter-story relative accelerations</u> as network inputs, avoiding the challenges of integrating acceleration responses (Contrary to ANN techniques requiring inter-story relative velocities and displacements.
- The performance of this IPV approach in determining the <u>existence</u>, <u>location</u>, and <u>extent</u> damage shows advantages over wavelet analysis





Background- Traditional Modeling Techniques-





Background- Traditional Modeling Techniques-





Background- Traditional Modeling Techniques-





Background- Traditional System Identification Techniques

- System ID is the process of building mathematical models of a dynamic system based on measured data.
- How It Is Done? By adjusting parameters within a given model until its output coincides as well as possible with the measured output.
- How Do We Know the Model is Good? Comparing the output of the model to measurements on data set that was not used for identification process.
- What Types of Models Can Be Used? Difference equation descriptions (ARX, ARMAX), all types of linear state-space models, black box nonlinear structures (Artificial Neural Networks), etc.
- Do We Have To Assume A Model for a Particular Type? For parametric models Yes.





### Background- Traditional System Identification Techniques

System ID consists of four basic steps:

- 1. Gathering experimental data
- 2. Choosing a set of candidate models
- 3. A criterion for selecting the best-fit model
- 4. An iterative optimization process, searching for the best-fit model.







Background- Traditional System Identification Techniques





2-SHM Research via Intelligent Parameter Varying (IPV) Artificial Neural Networks

Background- Traditional System Identification Techniques

**Parametric** 

Find "optimal functional representation" of the system using a "black box" model

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**Non-parametric** 

#### Background- Artificial Neural Networks (ANN)

- ANN has been inspired by the information processing in the brain.
  Because of their unique capabilities in nonlinear function approximation, ANN can be ideally suited for modeling and system ID.
- Literature shows how ANN can be effectively used for modeling, ID, and control of nonlinear dynamic systems.
- Nomenclature-
  - **Neuron** an information processing unit
  - Synapse of connecting path- connecting the neurons
  - Activation function- computes the output of a a neuron according to input activation level
  - Training set- Data used to train the network
  - Learning rule- algorithm used for testing the network
  - Testing set- data used for testing the network







Background-Artificial Neural Networks (ANN)





Background-Artificial Neural Networks (ANN)

Structure of a neural network





- Background-Artificial Neural Networks (ANN)
- i. Inspired by information processing in the brain
- ii. Learn from their environment

Hidden Layer



Output Layer



- Background-Artificial Neural Networks (ANN)
- i. Inspired by information processing in the brain
- ii. Learn from their environment
- iii. Radial Basis Functions Networks





Background-Artificial Neural Networks (ANN)



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Background-Artificial Neural Networks (ANN)

#### Black box" Approach

Typically, "Black box" neural networks are configured arbitrarily with a large number of system inputs and outputs, and are trained to provide the complete nonlinear mapping from the *m*dimensional input space to the *r*-dimensional output space.

When artificial neural networks are implemented using this approach, little (if any) of the system information that might be obtained from traditional modeling techniques is utilized.

Therefore, the associations between the neural network architecture and its weights to the underlying system dynamics and its parameters are rarely understood or utilized to improve the performance of the identification process.



Background-Artificial Neural Networks (ANN)

#### "Black box" Approach

Consider the most general form of a nonlinear plant with full state measurement:  $\dot{x} = f(x, u)$ 

y = x



Black box intelligent system identification provides a regression estimate of the entire plant dynamics using past sampled outputs and inputs.

Analogous to Prediction Error Methods, the network weights constitute a parameter vector that is iteratively modified, using the Training Set, in order to minimize differences between the predicted network output and the actual system response:

$$\varepsilon(t, w) = y(t) - \hat{y}(t \mid w)$$

The optimal set of network weights is thus the one that generates the smallest prediction error on unseen pairs of input-output measurements, the Testing Set, as quantified by a suitable scalar-valued error cost function.



#### Background-Artificial Neural Networks (ANN)

#### "Black box" Approach

Techniques that seek to minimize prediction error are collectively known as "backpropagation of error" methods, and have been discussed extensively in the literature (Haykin 1999). Defining a quadratic error cost function for *N* data points:

$$V_N(w) = \frac{1}{N} \sum_{t=1}^N \frac{1}{2} \varepsilon(t, w)^2$$

Analogous to the iterative parameter estimation techniques used in Prediction Error Methods, the most general backpropagation techniques consider the error cost function, its gradient, and its Hessian to update the network weights:

$$w(t+1) = w(t) - \mu \left[ R_N \right]^{-1} V'_N(w(t))$$

 $R_{\rm N}$  is a matrix that modifies the search direction. If selected to be the identity matrix, the backpropagation process is known as a Gradient Descent Method and if selected to be the Hessian matrix, it becomes Gauss-Newton Method.



Background-Artificial Neural Networks (ANN)

#### Intelligent Parameter Varying Approach

"Intelligent Parameter Varying" (IPV) approach to system identification incorporates artificial neural networks into a traditional parametric model, therefore combining the advantages of parametric models with the non-parametric capabilities of artificial neural networks.

Artificial neural networks are used to identify the nonlinear, timevarying portions of the system dynamics, that would be difficult to model using traditional approaches. The resulting model preserves a direct association between the neural network's architecture and its weights to the underlying system dynamics.


#### 2-SHM Research via Artificial Neural Networks

Consider a nonlinear system represented by the Linear Parameter Varying (LPV) model structure:

$$\dot{x} = f_1(x, u) \cdot x + f_2(x, u) \cdot u$$
$$y = x$$

Here, the model structure is derived using traditional modeling approaches, but  $f_1(x,u)$  and  $f_2(x,u)$  represent unknown constitutive nonlinearities.

The IPV approach introduced here would preserve the model structure without requiring a priori representations of the nonlinearities. Instead, these terms would be represented by separate artificial neural networks:  $g_1(x, u, w_1)$  and  $g_2(x, u, w_2)$ :  $\dot{x} = g_1(x, u, w_1) \cdot x + g_2(x, u, w_2) \cdot u$ y = x

By modeling the nonlinearities via separate artificial neural networks the relation between model structure and artificial neural networks parameters is preserved.

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Proposed Method-

Traditional Parametric Modeling Techniques

#### Intelligent Parameter Varying Technique

Traditional System ID Techniques

Artificial Neural Networks



	<b>Concept:</b>							
	Parametric	IP	V	Non-parametric				
	Combines the advantages of parametric and non- parametric techniques							
<i>x</i>	$= f_1(x, u) \cdot x + f_2(x, u)$	$u) \cdot u$	$\dot{x} = g_1(x, u)$	$(w_1) \cdot x + g_2(x, u, w_2)$	$) \cdot u$			
y	= x		v = x	62(4)492				
					LLU.S.E.			







**Implementation** 

Consider:

- i. A simple three-story "shear building" model
- ii. Elastic, Elasto-plastic, & Hysteretic restoring force models
- iii. Two damage mechanisms
- iv. El Centro 1940 earthquake ground acceleration and 3-Hz sinusoid base excitations









#### System Modeling

In accordance with Newton's 2<sup>nd</sup> law, the lateral equations of motion can be expressed:

$$-f_{3} - c_{3}(\dot{x}_{3} - \dot{x}_{2}) = m_{3}\ddot{x}_{3}$$
  

$$-f_{2} - c_{2}(\dot{x}_{2} - \dot{x}_{1}) + f_{3} + c_{3}(\dot{x}_{3} - \dot{x}_{2}) = m_{2}\ddot{x}_{2}$$
  

$$-f_{1} - c_{1}(\dot{x}_{1} - \dot{x}_{g}) + f_{2} + c_{2}(\dot{x}_{2} - \dot{x}_{1}) = m_{1}\ddot{x}_{1}$$

Where *m*'s represent the floor lumped masses, *c*'s are constant structural damping coefficients, and *f*'s are the inelastic stiffness restoring forces of the building.



#### System Modeling

Alternately, these state equations can be expressed in terms of story drifts

$$-f_{3} - c_{3}(\dot{u}_{3} - \dot{u}_{2}) - m_{3}\ddot{x}_{g} = m_{3}\ddot{u}_{3}$$
  
$$-f_{2} - c_{2}(\dot{u}_{2} - \dot{u}_{1}) + f_{3} + c_{3}(\dot{u}_{3} - \dot{u}_{2}) - m_{2}\ddot{x}_{g} = m_{2}\ddot{u}_{2}$$
  
$$-f_{1} - c_{1}(\dot{u}_{1} - \dot{u}_{g}) + f_{2} + c_{2}(\dot{u}_{2} - \dot{u}_{1}) - m_{1}\ddot{x}_{g} = m_{1}\ddot{u}_{1}$$

Where:

$$u_1 = x_1 - x_g$$
  $u_2 = x_2 - x_g$   $u_3 = x_3 - x_g$ 

Or in matrix form as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} = -\mathbf{M}\ddot{\mathbf{x}}_g - \mathbf{f}(\mathbf{x},\mathbf{u})$$





#### System Modeling

To facilitate these simulations, the shear-building model was parameterized using stiffness and yield displacement matrices *K* and *Xy*:

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \qquad \mathbf{X}_{y} = \begin{bmatrix} x_{y_{11}} & x_{y_{12}} \\ x_{y_{21}} & x_{y_{22}} \\ x_{y_{31}} & x_{y_{32}} \end{bmatrix}$$

The columns of the stiffness matrix represent the primary and secondary stiffnesses, while the rows correspond to building floors respectively. The columns and rows of the yield displacement matrix correspond similarly to the primary and secondary yield displacements and floors.



### **Restoring Force Model Elastic Elasto-plastic Hysteretic Restoring Force** $k_1$ Relative Displacement











#### **Simulations**

Simulation	Base Envitedier	Restoring Farrer Madal	Damage Mechanism	
Case	Excitation	Force Model		
I	3-Hz Sinusoid	. Elastic	The 1st damage mechanism.	
П	El Centro 1940			
Ш	3-Hz Sinusoid		The 2 <sup>nd</sup> damage mechanism followed by the 1 <sup>st</sup> damage	
IV	El Centro 1940		mechanism.	
V	3-Hz Sinusoid	Elasto-plastic	The 1st damage mechanism	1
VI	El Centro 1940			
VII	3-Hz Sinusoid	Hysteretic		
VIII	El Centro 1940	,		



#### **Networks Structures**

The stiffness and damping terms can be lumped together as net restoring forces:  $-R_2 - m_3 \ddot{x}_{\alpha} = m_3 \ddot{u}_3$ 

$$-R_{2} + R_{3} - m_{2}\ddot{x}_{g} = m_{2}\ddot{u}_{2}$$
$$-R_{1} + R_{2} - m_{1}\ddot{x}_{g} = m_{2}\ddot{u}_{2}$$

Using the IPV approach three separate RBFN networks were used to model these net restoring forces:

$$\hat{R}_{3} = g_{3}(\ddot{u}_{3}, \ddot{x}_{g})$$
$$\hat{R}_{2} = g_{2}(\ddot{u}_{2}, \ddot{x}_{g}, \hat{R}_{3})$$
$$\hat{R}_{1} = g_{1}(\ddot{u}_{1}, \ddot{x}_{g}, \hat{R}_{2})$$



#### **Networks Structures**

Literature abounds with variations of neural network architectures and activation functions for system identification; the most common architecture is a feedforward multi-layer network with hyperbolic tangent activation functions, the so-called Multi-Layer Perceptron (MLP) (Haykin 1999).

However, the Radial Basis Function Network (RBFN) may be better suited to the task of system identification for two reasons. First, the network uses multi-dimensional Gaussian (or radial basis) activation functions that, contrary to hyperbolic tangent functions, are localized with respect to the input space. As a result, parameter estimates obtained from a small region of the input space do not adversely affect estimates from other regions. Second, because the network output is a weighted sum of hidden layer outputs, the learning algorithm is very simple and computationally inexpensive.



**Networks Structures** 









**Networks Structures** 





#### **Networks Structures**

The inputs to each RBFN were normalized, and three activation functions were uniformly distributed along each dimension of the input space (-0.25, 0.50, and 1.25), resulting in 27 activation functions for each RBFN.

The standard deviations of each activation function were set to 0.5, and the weights were initially set to zero. These weights were updated incrementally using a Training Set consisting of randomly-selected inputoutput response data (50% of the entire simulation data).

Training continued until the change in the error cost function for a Testing Set (the remaining 50% of simulation data) fell below 1% over two consecutive iterations.

Learning rates of 20, 40 and 60 are used for the 1st, 2nd and 3rd floor restoring force networks, respectively.





A Few Important Criteria/Factors: For details see Saadat, 2003.

The IPV was applied to the acceleration responses of Cases I-VIII to identify the *Presence, Location*, and *Time of Dam*age.

- IPV incorporated RFBN into the parametric model (governing eqns of motion) to identify the restoring forces. The stiffness and damping were combined into net restoring forces R1, R2, R3
- For a single-output RFBN with N hidden layer neurons, there are 3 parameters that determine the network output: The network weights, w and the neuron centers c (N-element vectors), the neuron spreads, s.
- Input to each RBFN have different ranges, thus they were normalized and 3 basis functions were uniformly distributed along normalized dimensions of each input with centers located at –0.25, 0.5 and 1.25, resulting in 27 basis functions for each RBFN. The spread of each basis function was specified to be 5.0 (3 times the largest distance between basis functions)





Other Important Criteria/Factors: For details see Saadat, 2003.

- IPV identifies restoring forces and damage mechanisms <u>without a priori</u> <u>knowledge or assumptions regarding the constitutive characteristic</u>.
- Since the identified restoring forces are represented by and stored in its network weights, <u>all network weights</u>, <u>w</u>, were initialized to zero.
- To ensure that RFBNs properly generalized information from the acceleration responses, for each simulation case (I to VIII) the data (time samples of ground acceleration and resulting floor accelerations) was divided into <u>training</u> and <u>testing</u> (validation) sets.
- Each training data set consisted of one half of the simulated time history, each time sample selected randomly from the simulation data. The remaining half of time samples were randomly ordered and used to construct the testing (validation) data set.







Other Important Criteria/Factors: For details see Saadat, 2003.

- A standard back-propagation of error training algorithm, based on quadratic error cost function, was implemented for training. This training set was implemented in a systematic manner.
- Training continued until the error cost function, evaluated based on prediction errors from the testing data set, fell below 15 over two consecutive training epochs, where one epoch continue the use of all time samples in training data set to modify the network weights w.
- Samples of the "identified" restoring forces and "simulated" restoring forces for Cases I-VIII are presented in the following. The evolution force-displacement characteristics shows the <u>presence</u> of damage.
- To precisely isolate and identify the occurrence of damage and the time of damage, the IPV was implemented in a "snapshot mode," where response data was divided into 1-second time intervals and restoring forces were "identified."









Implementation of the radial basis functions networks



Implementation of the radial basis functions networks



#### Damage Quantification-

 Basic idea is monitoring the <u>change in the response of the</u> <u>structure, caused by a change in the restoring forces</u>. Thus, the percentage of the change in the response of the systems can be used to develop a measure for quantification of the damage.

#### Endurance Estimation-

 Basic idea is simulating the structure after occurrence of the damage with an excitation of x% of the original one in order to compare the new structural response against the pre-defined threshold.

Once the time and location of a damage in the system is determined one must *identify the severity* of the damage and consequently estimates the remaining life of the structure.











#### **Simulations**

Simulation	Base Envitedier	Restoring Farrer Madal	Damage Mechanism	
Case	Excitation	Force Model		
I	3-Hz Sinusoid	. Elastic	The 1st damage mechanism.	
П	El Centro 1940			
Ш	3-Hz Sinusoid		The 2 <sup>nd</sup> damage mechanism followed by the 1 <sup>st</sup> damage	
IV	El Centro 1940		mechanism.	
V	3-Hz Sinusoid	Elasto-plastic	The 1st damage mechanism	1
VI	El Centro 1940			
VII	3-Hz Sinusoid	Hysteretic		
VIII	El Centro 1940	,		



Estimated restoring forces and **Damage Propagation** 

5 3<sup>rd</sup> Floor Case I 0 3-Hz sinusoid -5 -2 0 2 -3 -1 Elastic 10 2<sup>nd</sup> Floor 5 0 -5 -10 -5 -3 -2 2 5 0 3 10 1<sup>st</sup> Floor 5 0 -5 -10 -2 0 2 -6 4 6 Relative Displacement (mm)

Restoring force initially elastic and remains elastic but "softens"



1-Second "snapshot" of the "estimated" restoring forces. *Time of damage occurrence*. Changes in restoring force characteristics readily identified.



Estimated restoring forces and Damage Propagation



Restoring force initially elastic but severely softens and plastic deformation



\*\*\*\*

1-Second "snapshot" of the "estimated" restoring forces. *Time of damage occurrence*. Changes in restoring force characteristics readily identified.



Figure 4.15: Snapshots of identified restoring forces (N) for Case IV: (a) vs. relative displacement, (b) vs. time, before and after damage (o) , 3.5-4.5 seconds.



onds.

Estimated restoring forces and Damage Propagation



Primary column stiffness already reduced due to previous structural damage, and excursion into hysteretic behavior.


1-Second "snapshot" of the "estimated" restoring forces. *Time of damage occurrence*. Changes in restoring force characteristics readily identified.



Shows additional softening from structural damage at 14.004 and 14.142 seconds, and hysteretic degradation/damage.



**Effects of ANN parameters on the IPV Technique** 

- Gaussian basis functions parameters
  - i. Number
  - ii. Location, and
  - iii. Spread of the basis functions
- IPV accuracy is measured by
  - i. Error cost function on the test set, and
  - ii. Total number of the training epochs





#### **Basis Functions**

Local: their support covers a small range of input space giving weighted local averages in input space.





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#### **Basis Functions**

Global: their support covers a large range of input space allowing the dispersion of properties of the estimated function.



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#### **Basis Functions**

**Location of the basis functions:** 

#### – Fixed:

 the basis functions can be chosen in a random or uniform fashion from the input data space depending on the distribution characteristics of data.

#### - Float:

 the basis functions can be chosen in a selforganized fashion or supervised learning process.





#### **Learning Algorithms**

Neural networks can be classified based upon Learning algorithm used for updating the weights,

- Error-Correction Learning
- Hebbian Learning
- Competitive Learning
- Boltzman Learning
- Supervised Learning
- Reinforcement Learning
- Unsupervised Learning
- And more





A Total of 1792 simulations were conducted (224 for each of the 8 cases)

- Simulations for each case arranged into two sets according to the no. of Gaussian basis functions for the networks applied for each floor.
- For the 1st & 2nd simulation sets 2 and 3 basis functions used along normalized input space dimensions, resulting in a total of 23=8 and 33=27 basis functions for each network, respectively.
- Each group was divided into 4 groups according to the Gaussian basis function locations, where each group consisted of 28 simulations for different Gaussian basis function spreads, ranging from 0.0625 to 40.0.







### Number and location of Gaussian basis functions for Simulation Set 1, Groups I to IV.















20 locations of Gaussian basis functions for Simulation Set 2, Groups I to IV.

$$3^3 = 27$$















10

**Effects of basis function spreads on error cost function, Simulation Case III, for Simulation Set 1 and Set 2** 

 $10^{4}$ 

 $10^{2}$ 10 Error Cost Function on the Test Set Error Cost Function on the Test Set 10<sup>0</sup> 10<sup>0</sup> Case VIII 10<sup>-2</sup> 10 El Centro 1940 10<sup>-4</sup> 10<sup>-4</sup> Hysteretic 10 10<sup>-6</sup> (a): 2 basis functions (b): 3 basis functions 10 10 0 10 20 30 40 10 20 30 40 σ (a)

Decreasing/Increasing in error cost function as Gaussian basis function spreads increase.

### **Total number of the training epochs**

Effects of basis function spreads on total number of training epochs, Simulation Case III, Simulations Sets 1 and 2.

Case VIII

El Centro 1940

Hysteretic

(a): 2 basis functions

(b): 3 basis functions



10

Decreasing/increasing in computational time due to dependency on Gaussian basis function spreads.

 $10^{3}$ 



Spread of Gaussian basis functions are important- Gaussian basis function over the normalized input with different spreads, located at 0, 0.5 and 1.



Weight distribution of the radial basis functions networks for Simulation Case VIII. 2 Gaussian basis functions at 0 and 1.



 $2^3 = 8$ 

Case VIII El Centro 1940 Hysteretic



Weight distribution of the radial basis functions networks for Simulation Case VIII: 2 Gaussian basis functions at 0 and 1.



 $2^3 = 8$ 

Case VIII El Centro 1940 Hysteretic

 $\sigma = 10$ 



Weight distribution of the radial basis functions networks for Simulation Case VIII: 3 Gaussian basis functions at 0, 0.5 and 1.







Weight distribution of the radial basis functions networks for Simulation Case VIII: 3 Gaussian basis functions at 0, 0.5 and 1.



 $3^3 = 27$ Case VIII El Centro 1940 Hysteretic

 $\sigma = 10$ 

These results suggested selection of

- Two Gaussian basis functions
- Located at the limits of normalized input range, with
- Spreads of 10







Estimated restoring forces- with changes to the no. of Gaussian basis functions and their spread.





1-second "snap shot" of the estimated restoring forces

Case I 3-Hz sinusoid Elastic 3.5 – 4.5 sec (a): Vs. Disp. (b): Vs. Time





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#### **Estimated restoring forces**

Case VIII

Hysteretic



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1-second "snap shot" of the estimated restoring forces

Case VIII El Centro 1940 Hysteretic 13.5 – 14.5 sec (a): Vs. Disp. (b): Vs. Time





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**Improvement in IPV accuracy and required computational time.** Arbitrary RFBN parameters: <u>Blue bar</u>; Optimized RFBN parameters: <u>Red bars.</u>





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**Effects of Measurement Noise on the IPV Technique** 

Noise is an undesired disturbance within the frequency band of interest, introduced by man-made and natural sources that distort the information carried by the signal. The following equation is used wh identifying signals subject to measurement noise.

$$SNR = 10 \cdot \log \left( \frac{\text{Peak Signal Power}}{\text{Total Noise Power}} \right)$$

 $6.02\,(dB) \leq SNR \leq 40.0\,(dB)$ 



#### **Estimated restoring forces in a noisy environment**



Case VIII El Centro 1940 Hysteretic SNR = 6.02 (dB)



### **Estimated restoring forces**

Case VIII El Centro 1940 Hysteretic SNR = 40.0 (dB)





### 1-second "snap shot" of the estimated restoring forces





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**Effects of Measurement Noise on the IPV Technique** 

• What SNR level is acceptable?

Case VIII El Centro 1940 Hysteretic



Cross-Correlation factors between identified and actual values of net restoring forces vs SNR: Case VIII.



## **Conclusions and Summary**

### A simple yet powerful technique that can

- 1. Identify the instance of the damage occurrence
- 2. Identify the substructure where the damage is located
- 3. Quantify the extent of damage, from system parameters
- 4. Estimate the remaining life of the structure
- 5. Can be implemented solely in time domain
- 6. Can be applied to non-linear time-varying systems
- 7. Be robust with respect to measurement noise
- 8. Can be tailored/parameterized accordingly (Model/System ID Based)
- 9. Be used in both deterministic & broadband/random excitation cases, & the instance of damage occurrence & the extent of change in system parameters can be detected and be quantified.







## **Conclusions and Future Research Directions**

- A simple yet powerful technique that overcomes the intrinsic limitations of traditional techniques is developed- This powerful method can be further improved by:
  - 1. Use of different basis functions
  - 2. Use of different learning algorithms
  - 3. Use of data compression techniques
  - 4. Study the effects of missing data
  - 5. Defining performance matrices and proper damage indices
  - 6. Experimental verification







#### **Experimental Set Up- The Prototype Structure**



Figure 3 Experimental shear building model.





**Analytical Model of the Structure – Shear Beam** 



Figure 1 Lamped-mass, shear-building model of a three-story base excited structure.



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Figure 12 (a) Third floor static lateral loading, (b) second floor static lateral loading, and (c) first floor static lateral loading.







Figure 4 Schematic of the experimental set up.



### Measuring Inter-story Displ - Identifying Restoring Force Profiles a) Free-vibration tests.





0.4

0.4

0.4

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### **Measuring Inter-story Accel - Identifying Restoring Force Profiles** b) Harmonic Excitation tests.



Figure 9 Harmonic excitation: (a) filtered acceleration responses and (b) identified restoring force profiles.



**Identifying Restoring Force Profiles & Structural Damage** c) Damage between the base & 1<sup>st</sup> floor; Before & After Damage.



Figure 14 Identified restoring force profiles associated with harmonic excitation and structural damage between the base and first floors: (a) 1 s before damage and (b) 1 s after damage.

#### Under Harmonic Excitation


## **Experimental Verification of Intelligent Parameter Varying (IPV) Technique**

**Identifying Restoring Force Profiles & Structural Damage** d) Damage between the 1<sup>st</sup> & 2<sup>nd</sup> floors; Before & After Damage.



Figure 15 Identified restoring force profiles associated with harmonic excitation and structural damage between the first and second floors: (a) 1 s before damage and (b) 1 s after damage.

#### Under Harmonic Excitation



## **Experimental Verification of Intelligent Parameter Varying (IPV) Technique**

**Identifying Restoring Force Profiles & Structural Damage** e) Damage between the 1<sup>st</sup> & 2<sup>nd</sup> floors; Before & After Damage.



Figure 17 Identified restoring force profiles associated with earthquake excitation and structural damage between the first and second floors: (a) 1 s before damage and (b) 1 s after damage.

### Under Earthquake (Broad-Band) Excitation



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## **Other Future/Emerging Research Directions**

- Thoroughly evaluate merits and limitations of Wavelet, IPV, and EMD, in their application for on-line SHM in a distributed sensor network environment and for local area damage detection with limited sensors.
- Thoroughly evaluate the performance of these techniques on various damage severity levels; damage allocation; in random environment and on-line implementation.
- Quantify and develop an integrated system that can self select what diagnosis system to use based on the type of data, availability of the model, type of sensor, UQ and reliability.
- Develop and evaluate the merits and limitations of Support Vector Machines and Artificial Immune Systems for SHM.
- Carry out a thorough study on the use of SHM strategies for Uncertainty Quantification







# ADDITIONLA SLIDES ON SPECIFIC RECOMMENDATIONS







## **Emerging Research Directions: UQ and SHM**



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UQ is critical for both system design and maintenance decisions.

## **Emerging Research Directions: SVM for SHM**



The SVM hyper plane separating the data in undamaged and damaged classes





The adaptive SVM hyper lines indicate the health condition of a structure to assess its safety margin



SVM can simultaneously assess the structural reliability and UQ.

# **Emerging Research Directions: SVM for SHM**



SVM can assess limit states and reliability. Both are important in SHM



## **Emerging Research Directions: AIS for SHM**

- Artificial Immune System is a computational model based on the self/ non-self discrimination process performed by the T-cells in natural immune systems
- Uses a Negative Selection Algorithm (NSA). Has proved top be very effective, accurate and reliable in several applications, such as computer network security analysis, fault detection, etc.
- In biomedical data classification. AIS has been "trained" to detect the "normal" vs "carrier" data with high accuracy. Difficult to distinguish.
- Some result have been shown for SHM by Bo Chen, et al.





